Study of Pure Annihilation Decays $B_{d,s} \to D^0 \overline{D}^0$

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Abstract

With heavy quark limit and hierarchy approximation $\lambda_{QCD} \ll m_D \ll m_B$, we analyze the $B \to D^0\overline{D}^0$ and $B_s \to D^0\overline{D}^0$ decays, which occur purely via annihilation type diagrams. As a roughly estimation, we calculate their branching ratios and CP asymmetries in Perturbative QCD approach. The branching ratio of $B \to D^0\overline{D}^0$ is about 3.8×10^{-5} that is just below the latest experimental upper limit. The branching ratio of $B_s \to D^0\overline{D}^0$ is about 6.8×10^{-4}, which could be measured in LHC-b. From the calculation, it could be found that this branching ratio is not sensitive to the weak phase angle γ. In these two decay modes, there exist CP asymmetries because of interference between weak and strong interaction. However, these asymmetries are too small to be measured easily.

1 Introduction

In the Standard Model (SM), CP-violation (CPV) arises from a complex phase in the Cabibbo-Kobayashi- Maskawa (CKM) quark mixing matrix, and the angles of unitary triangle are defined as [1]:

$$\beta = \arg\Big[-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\Big], \quad \alpha = \arg\Big[-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\Big], \quad \gamma = \arg\Big[-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\Big]. \tag{1}$$

In order to test SM and search for new physics, many measurements of CP-violation observables can be used to constrain these above angles. It is well known that we measure β precisely using the golden decay mode $B \to J/\psi K_s$; the angle α can be determined with decay $B \to \pi\pi$ and γ could be measured precisely in Large Hadron Collider (LHC) with decay mode $B_s \to D_s K$.

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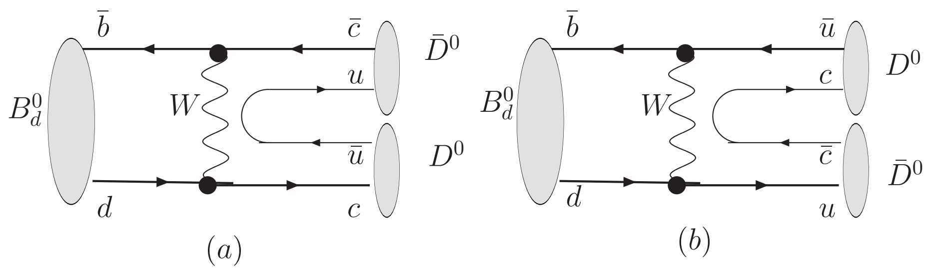

Figure 1: The quark level Feynman diagrams for $B_d \to D^0\bar{D}^0$ process

Besides the above channels mentioned, many other channels are used to cross check the measurements. Among these decays, $B \to DD$ decay is considered to test the β measurement. For $B \to DD$ decay, the analysis based on $SU(3)$ symmetry [2], iso-spin symmetry [3] and factorization approach [4] have been done in last several years. However, the the calculation of decay $B^0 \to D^0\overline{D}^0$ has difficulties. It is a pure-annihilation diagram decay, also named W-exchange diagram decay, which is power suppressed in factorization language. The quark diagrams of this decay are shown in Figure 1. Theoretically, QCD factorization approach (QCDF) [5] and soft collinear effective theory (SCET)[6] can not deal decays with two heavy charmed mesons effectively. In Ref.[7, 8], perturbative QCD (PQCD) has been exploited to B meson decays with one charm meson in the final states and the results agree with experimental data well. Specially, the pure annihilation type B decays with charmed meson were studied in Ref.[8].

In the standard model picture, the W boson exchange causes $\bar{b}d \to \bar{c}c$, and the $\bar{u}u$ quarks are produced from a gluon. This gluon attaches to any one of the quarks participating in the W boson exchange. In decay $B \to D^0\overline{D}^0$, the momentum of the final state D meson is $\frac{1}{2}m_B(1-2r^2)$, with $r = m_D/m_B$. If we consider heavy quark limit and hierarchy approximation $\lambda_{QCD} \ll m_D \ll m_B$, the D meson momentum is nearly $m_B/2$. According to the distribution amplitude used in Ref.[7], the light quark in D meson carrying nearly 40% of the D meson momentum. So, this light quark is still a collinear quark with 1 GeV energy, like that in $B \to DM$ [7, 8], $B \to K(\pi)\pi$ [9, 10] decays. The gluon could be viewed as a hard gluon approximatively, so we can treat the process perturbatively where the four-quark operator exchanges a hard gluon with $u\bar{u}$ quark pair. Of course, we are able to calculate the diagrams if charm quark and up quark exchange. As a roughly estimation, we give the branching ratio and CP-violation of $B_{d,s} \to D^0\overline{D}^0$.

In this article, the analytic formulas for the decay amplitudes will be shown in the next section. In section 3, we give the numerical results and summarize this article in section 4.

2 Analytic formulas

For simplicity, we set B meson at rest in our calculation. In light-cone coordinates, the momentum of B, D^0 and$\overline{D}^0$ are:

$$P_B = \frac{M_B}{\sqrt{2}}(1, 1, \vec{0}); P_2 = \frac{M_B}{\sqrt{2}}(1 - r^2, r^2, \vec{0}); P_3 = \frac{M_B}{\sqrt{2}}(r^2, 1 - r^2, \vec{0}). \quad (2)$$

we define the light (anti-)quark momenta in B, D^0 and $\overline{D}^0$ mesons as k_1, k_2, and k_3 as:

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \quad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \quad (3)$$

In PQCD, we factorize the decay amplitude into soft(Φ), hard(H), and harder (C) dynamics characterized by different scales, [9, 10]

$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \mathrm{Tr}\Big[C(t)\Phi_B(x_1, b_1)\Phi_D(x_2, b_2)\Phi_D(x_3, b_3)H(x_i, b_i, t)S_t(x_i)\, e^{-S(t)}\Big]. \quad (4)$$

In above equation, b_i is the conjugate space coordinate of the transverse momentum $\mathbf{k}_{iT}$, and t is the largest energy scale. C is Wilson coefficient, and Φ is the wave function. The last term, $e^{-S(t)}$, contains two kinds of contributions. One is due to the resummation of the large double logarithms from renormalization of ultra-violet divergence $\ln tb$, the other is from resummation of double logarithm $\ln^2 b$ from the overlap of collinear and soft gluon corrections, which is called Sudakov form factor. The hard part H can be calculated perturbatively, and it is channel dependent. More explanation of above formula and review about PQCD can be found in many reference, such as [9, 10, 11].

As a heavy meson, the B meson wave function is not well defined, neither is D meson. In heavy quark limit, we take them as:

$$\Phi_B(x, b) = \frac{i}{\sqrt{6}}\left[\not{P} + M_B\right]\gamma_5 \phi_B(x, b), \quad (5)$$

$$\Phi_D(x, b) = \frac{i}{\sqrt{6}}\gamma_5\left[\not{P} + M_D\right]\phi_D(x, b). \quad (6)$$

The Lorentz structure of two mesons are different because the B meson is initials state and D meson is final state.

The effective Hamiltonian $\bar{b} \to \bar{q}(q = d, s)$ is given by [12]:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\Big\{V_{cq}V_{cb}^*\Big[C_1(\mu)O_1^c(\mu) + C_2(\mu)O_2^c(\mu)\Big] + V_{uq}V_{ub}^*\Big[C_1(\mu)O_1^u(\mu) + C_2(\mu)O_2^u(\mu)\Big]$$
$$- V_{tb}^*V_{tq}\sum_{i=3}^{10} C_i(\mu)O_i(\mu)\Big\}, \quad (7)$$

where $C_i(\mu)(i=1,\cdots,10)$ are Wilson coefficients at the renormalization scale μ and the four quark operators $O_i(i=1,\cdots,10)$ are

$$\begin{array}{ll}
O_1^c=(\bar{b}_i c_j)_{V-A}(\bar{c}_j q_i)_{V-A}, & O_2^c=(\bar{b}_i c_i)_{V-A}(\bar{c}_j q_j)_{V-A},\\
O_1^u=(\bar{b}_i u_j)_{V-A}(\bar{u}_j q_i)_{V-A}, & O_2^u=(\bar{b}_i u_i)_{V-A}(\bar{u}_j q_j)_{V-A},\\
O_3=(\bar{b}_i q_i)_{V-A}\sum_q(\bar{q}_j q_j)_{V-A}, & O_4=(\bar{b}_i q_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A},\\
O_5=(\bar{b}_i q_i)_{V-A}\sum_q(\bar{q}_j q_j)_{V+A}, & O_6=(\bar{b}_i q_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A},\\
O_7=\frac{3}{2}(\bar{b}_i q_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V+A}, & O_8=\frac{3}{2}(\bar{b}_i q_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A},\\
O_9=\frac{3}{2}(\bar{b}_i q_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V-A}, & O_{10}=\frac{3}{2}(\bar{b}_i q_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A}.
\end{array} \tag{8}$$

Here i and j are $SU(3)$ color indices; in $O_{3,\ldots,10}$ the sum over q runs over the quark fields that are active at the scale $\mu=O(m_b)$, i.e., $q\in\{u,d,s,c,b\}$. For Wilson coefficients, we will also use the leading logarithm summation for QCD corrections, although the next-to -leading order calculation already exists [12]. This is the consistent way to cancel the explicit μ dependence in the theoretical formulae.

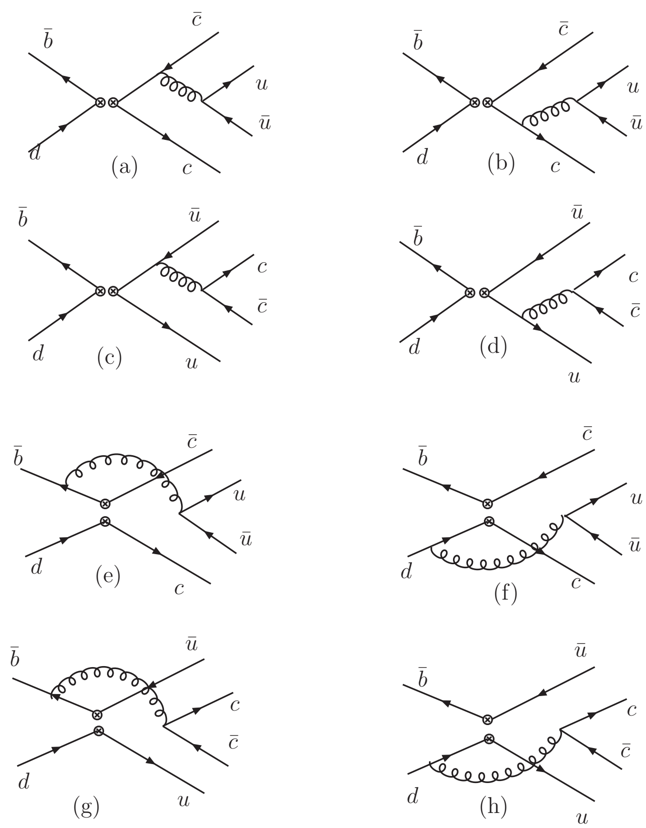

Figure 2: The leading order Feynman diagrams for $B_d\to D^0\overline{D}^0$ process in PQCD approach

According to the effective Hamiltonian in eq.(7, 8), the lowest order diagrams of $B\to D^0\overline{D}^0$ are

drawn in Fig. 2. We first calculate the usual factorizable diagrams (a), (b), (c) and (d). For the $(V-A)(V-A)$ operators, their contributions of (a) and (c) are always canceled by diagrams (b) and (d) respectively because of current conservation. For the $(V-A)(V+A)$ operators, these diagrams can not give contribution, either. That's to say, factorizable diagrams have no contribution. For non-factorizable diagrams (e), (f), (g) and (h), we find the hard part for $(V-A)(V-A)$ operators are same to $(V-A)(V+A)$ operators. We group the contribution of diagrams (e) and (f), denoted by M_a, as follows:

$$M_a[C_i] = \frac{64\pi C_F M_B^2}{\sqrt{2N_C}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1\, b_2 db_2\ \phi_B(x_1,b_1)\phi_D(x_2,b_2)\phi_D(x_3,b_2) \times \Big\{ \Big[x_1 + x_2 + (2x_3 - x_2)r^2\Big] C_i(t_a^1)E(t_a^1)h_a^{(1)}(x_1,x_2,x_3,b_1,b_2) + \Big[-x_3 + (2x_1 - 2x_2 + x_3)r^2\Big] C_i(t_a^2)E(t_a^2)h_a^{(2)}(x_1,x_2,x_3,b_1,b_2)\Big\}, \quad (9)$$

where $C_F = 4/3$ is the group factor of $\mathrm{SU}(3)_c$ gauge group, and C_i is Wilson coefficient. The function E_m is defined as

$$E(t) = \alpha_s(t)\, e^{-S_B(t)-S_D(t)-S_D(t)}, \quad (10)$$

and S_B, S_D result from Sudakov factor and single logarithms due to the renormalization of ultra-violet divergence. The functions h_a is the Fourier transformation of virtual quark and gluon propagators. It is defined by

$$h_a^{(j)}(x_1,x_2,x_3,b_1,b_2) = \Big\{ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{x_2x_3(1-2r^2)}\, b_1)\mathrm{J}_0(M_B\sqrt{x_2x_3(1-2r^2)}\, b_2)\theta(b_1-b_2) + (b_1 \leftrightarrow b_2)\Big\} \times \begin{pmatrix} \mathrm{K}_0(M_B F_{a(j)} b_1), & \text{for} \quad F^2_{a(j)} > 0 \\ \frac{\pi i}{2}\mathrm{H}_0^{(1)}(M_B\sqrt{|F^2_{a(j)}|}\ b_1), & \text{for} \quad F^2_{a(j)} < 0 \end{pmatrix}, \quad (11)$$

with:

$$F^2_{a(1)} = -x_1 - x_2 - x_3 + x_1x_3 + x_2x_3 + (x_2 + x_3 - x_1x_3 - 2x_2x_3)r^2; \quad (12)$$

$$F^2_{a(2)} = x_2x_3 - x_1x_3 + (x_1x_3 - 2x_2x_3)r^2. \quad (13)$$

In above equation, $\mathrm{H}_0^{(1)}(z) = \mathrm{J}_0(z) + i\,\mathrm{Y}_0(z)$. In order to reduce the large logarithmic radiative corrections, the hard scale t in the amplitudes is selected as the largest energy scale in the hard part:

$$t_a^j = \max(M_B\sqrt{|F^2_{a(j)}|}, M_B\sqrt{(1-2r^2)x_2x_3}, 1/b_1, 1/b_2). \quad (14)$$

Analogically, we can get the M_b, which comes from the contribution of diagrams (g) and (h):

$$
\begin{aligned}
M_b[C_i] = \frac{64\pi C_F M_B^2}{\sqrt{2N_C}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1\, b_2 db_2\ \phi_B(x_1,b_1)\phi_D(x_2,b_2)\phi_D(x_3,b_2) \\
\times \Big\{ \Big[1 - x_3 + (2+2x_1-2x_2+x_3)r^2\Big] C_i(t_b^1)E(t_b^1)h_b^{(1)}(x_1,x_2,x_3,b_1,b_2) \\
+ \Big[x_1 + x_2 - 1 + (-2 - x_2 + 2x_3)r^2\Big] C_i(t_b^2)E(t_b^2)h_b^{(2)}(x_1,x_2,x_3,b_1,b_2)\Big\},
\end{aligned}
\quad (15)
$$

and the functions are defined as:

$$
\begin{aligned}
& h_b^{(j)}(x_1,x_2,x_3,b_1,b_2) = \\
& \Big\{ \frac{\pi i}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{1-x_2-x_3+x_2x_3+(x_2+x_3-2x_2x_3)r^2}\, b_1) \\
& \quad \times \mathrm{J}_0(M_B\sqrt{1-x_2-x_3+x_2x_3+(x_2+x_3-2x_2x_3)r^2}\, b_2)\theta(b_1-b_2) \\
& \quad + (b_1 \leftrightarrow b_2)\Big\} \times \begin{pmatrix} \mathrm{K}_0(M_B F_{b(j)} b_1), & \text{for} \quad F_{b(j)}^2 > 0 \\ \frac{\pi i}{2}\mathrm{H}_0^{(1)}(M_B\sqrt{|F_{b(j)}^2|}\ b_1), & \text{for} \quad F_{b(j)}^2 < 0 \end{pmatrix};
\end{aligned}
\quad (16)
$$

$$
F_{b(1)}^2 = -1 - x_1x_3 + x_2x_3 + (x_1x_3 - 2x_2x_3)r^2, \quad (17)
$$

$$
F_{b(2)}^2 = 1 - x_1 - x_2 - x_3 + x_1x_3 + x_2x_3 + (x_2 + x_3 - x_1x_3 - 2x_2x_3)r^2, \quad (18)
$$

$$
t_b^j = \max(M_B\sqrt{|F_{b(j)}^2|}, M_B\sqrt{1-x_2-x_3+x_2x_3+(x_2+x_3-2x_2x_3)r^2}, 1/b_1, 1/b_2). \quad (19)
$$

So, the decay amplitude of decay $B_d \to D^0\overline{D}^0$ can be read as:

$$
\begin{aligned}
\mathcal{A}_1 &= V_{cb}^* V_{cd} M_a[C_2] - V_{tb}^* V_{td} M_a[C_5 + C_7] + V_{ub}^* V_{ud} M_b[C_2] - V_{tb}^* V_{td} M_b[C_5 + C_7] \\
&= V_{cb}^* V_{cd} T_1 - V_{tb}^* V_{td} P_1 \\
&= V_{tb}^* V_{td} P_1 (1 + z_1 e^{i(\beta+\delta_1)}),
\end{aligned}
\quad (20)
$$

where β is weak phase angle defined in Eq.(1), and δ_1 is the strong phase, which plays an important role in studying CP-violation. In above calculation, we denote that

$$
\begin{aligned}
T_1 &= M_a[C_2] - M_b[C_2], \\
P_1 &= M_a[C_5 + C_7] + M_b[C_5 + C_7] + M_b[C_2],
\end{aligned}
\quad (21)
$$

and

$$
z_1 = \left|\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right| \left|\frac{T_1}{P_1}\right|, \quad (22)
$$

which describes the ratio between tree diagram and penguin diagram. The corresponding charge conjugate decay is

$$\overline{\mathcal{A}_1} = V_{tb}V_{td}^* P_1(1 + z_1 e^{i(-\beta+\delta_1)}). \tag{23}$$

Therefore, the averaged decay width Γ for $B^0 \to D^0\overline{D}^0$ decay is then given by

$$\Gamma(B^0 \to D^0\overline{D}^0) = \frac{G_F^2 M_B^3}{128\pi}(1-2r^2)|V_{tb}^* V_{td} P_1|^2 \big|1 + z_1^2 + 2z_1 \cos\beta\cos\delta_1|. \tag{24}$$

¿From this equation, we know that the averaged branching ratio is a function of CKM angle β, if $z_1 \neq 0$. Derived from Eq.(20) and Eq.(23), the direct CP-violation can be formulated as:

$$A_{CP}^{dir}(B \to D^0\overline{D}^0) = \frac{|A_{B_d\to D^0\overline{D}^0}|^2 - |A_{\overline{B}_d\to\overline{D}^0 D^0}|^2}{|A_{B_d\to D^0\overline{D}^0}|^2 + |A_{\overline{B}_d\to\overline{D}^0 D^0}|^2} = \frac{-2z_1\sin\beta\sin\delta_1}{1+z_1^2+2z_1\cos\beta\cos\delta_1}. \tag{25}$$

For $B_s^0 \to D^0\overline{D}^0$ and its conjugate decay, we write the decay amplitudes and rearrange them as:

$$\begin{aligned}
\mathcal{A}_2 &= V_{cb}^*V_{cs}M_a[C_2] - V_{tb}^*V_{ts}M_a[C_5+C_7] + V_{ub}^*V_{us}M_b[C_2] - V_{tb}^*V_{ts}M_b[C_5+C_7] \\
&= V_{ub}^*V_{us}M_b[C_2] - V_{tb}^*V_{ts}\Big\{M_a[C_5+C_7] + M_b[C_5+C_7] - \frac{V_{cb}^*V_{cs}}{V_{tb}^*V_{ts}}M_a[C_2]\Big\} \\
&= V_{ub}^*V_{us}T_2 - V_{tb}^*V_{ts}P_2 \\
&= V_{ub}^*V_{us}T_2\Big[1 + z_2 e^{i(-\gamma+\delta_2)}\Big], \qquad (26) \\
\overline{\mathcal{A}_2} &= V_{ub}V_{us}^*T_2\Big[1 + z_2 e^{i(\gamma+\delta_2)}\Big], \qquad (27)
\end{aligned}$$

where T_2, P_2 and z_2 are defined as:

$$\begin{aligned}
T_2 &= M_b[C_2], \\
P_2 &= M_a[C_5+C_7] + M_b[C_5+C_7] - \frac{V_{cb}^*V_{cd}}{V_{tb}^*V_{ts}}M_a[C_2], \\
z_2 &= \left|\frac{V_{tb}^*V_{ts}}{V_{ub}^*V_{us}}\right|\left|\frac{T_2}{P_2}\right|. \qquad (28)
\end{aligned}$$

So, the averaged decay width and direct CP violation can be formulated as:

$$\begin{aligned}
\Gamma(B_s \to D^0\overline{D}^0) &= \frac{G_F^2 M_B^3}{128\pi}(1-2r^2)\big|V_{ub}V_{us}^*T_2\big|^2(1+z_2^2+2z_2\cos\delta_2\cos\gamma), \qquad (29) \\
A_{CP}^{dir}(B_s \to D^0\overline{D}^0) &= \frac{|A_{B_s\to D^0\overline{D}^0}|^2 - |A_{\overline{B}_s\to\overline{D}^0 D^0}|^2}{|A_{B_s\to D^0\overline{D}^0}|^2 + |A_{\overline{B}_s\to\overline{D}^0 D^0}|^2} = \frac{2z_2\sin\gamma\sin\delta_2}{1+z_2^2+2z_2\cos\gamma\cos\delta_2}. \qquad (30)
\end{aligned}$$

In our calculation, we set $m_c \approx m_D$, just because $m_D - m_c \approx \Lambda_{QCD}$ and $\frac{\Lambda_{QCD}}{m_B} \to 0$ in the heavy quark limit.

Table 1: Amplitudes (10^{-3} GeV) of $B_d \to D^0\overline{D}^0$ and $B_s \to D^0\overline{D}^0$.

	$B_d \to D^0\overline{D}^0$	$B_s \to D^0\overline{D}^0$
$T(e)+T(f)$	$68+17\,i$	$66+27\,i$
$P(e)+P(f)$	$0.80+0.23\,i$	$0.77+3.68\,i$
$T(g)+T(h)$	$9.81-2.99\,i$	$14.0-0.6\,i$
$P(g)+P(h)$	$0.08-0.02\,i$	$-0.01+0.01\,i$

3 Numerical Results

For B meson, the distribution amplitude is well determined by charmless B decays [9, 10], which is chosen as

$$\phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{M_B^2\, x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \tag{31}$$

with parameters $\omega_b = 0.4$ GeV, and $N_B = 91.745$ GeV which is the normalization constant using $f_B = 190$ MeV. For B_s meson, we use the same wave function according to SU(3) symmetry, where $\omega_b = 0.4$ GeV, $N_{B_s} = 119.4$ GeV and $f_{B_s} = 230$ MeV.

Since the c quark is much heavier than the u quark, the c quark shares more momentum, and this function should be asymmetric with respect to $x = 1/2$. The asymmetry is parameterized by a_D. Similar to the b-dependence on the wave function of B meson, for controlling the size of charmed mesons, we also introduce the intrinsic b-dependence on those of charmed mesons. Hence, we use the wave function of D meson as [13]

$$\phi_D(x,b) = \frac{3}{\sqrt{2N_c}} f_D x(1-x)\Big[1 + a_D(1-2x)\Big]\exp\left[-\frac{1}{2}(\omega_D b)^2\right]. \tag{32}$$

We use $a_D = 0.7$ and $\omega_D = 0.4$ in above function. Other parameters, such as meson mass, decay constants, the CKM matrix elements and the lifetime of B meson are list [1, 14]:

$$\begin{gathered}
M_B = 5.28 \text{ GeV},\ M_{B_S} = 5.36 \text{ GeV},\ M_D = 1.87 \text{ GeV},\ f_D = 210 \text{ MeV}, \\
|V_{ud}| = 0.974,\ |V_{ub}| = 4.3\times 10^{-3},\ |V_{cd}| = 0.23,\ |V_{cb}| = 41.6\times 10^{-3} \\
|V_{td}| = 7.4\times 10^{-3},\ |V_{tb}| = 1.0,\ |V_{us}| = 0.226,\ |V_{cs}| = 0.957, \\
|V_{ts}| = 41.6\times 10^{-3}, \quad \tau_{B_d^0} = 1.54\times 10^{-12} \text{ s}, \quad \tau_{B_s^0} = 1.46\times 10^{-12} \text{ s}.
\end{gathered} \tag{33}$$

With these parameters fixed, we calculate the decay amplitudes of the $B^0 \to D^0\overline{D}^0$ and $B_s \to D^0\overline{D}^0$ decays in Table 1. From the table, we notice that the main contribution comes from the tree

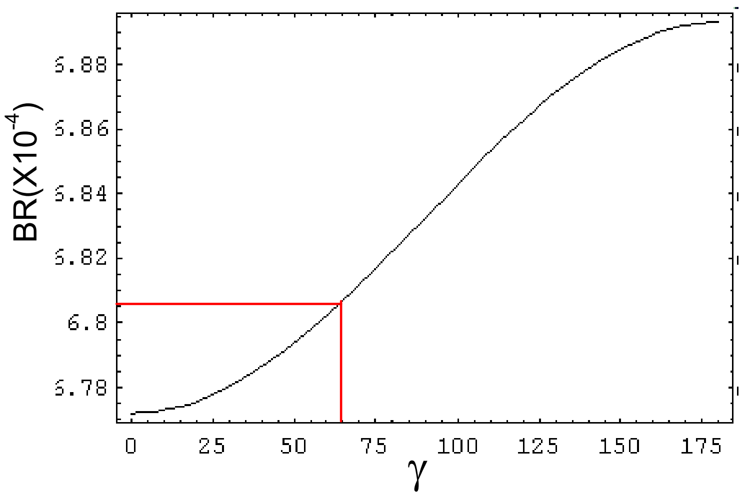

Figure 3: The branching ratio of $B_s \to D^0\overline{D}^0$ changes with CKM angle γ.

diagram (e) and (f). And our predictions for the branching ratio of each mode corresponding to $\beta = 23°$ and $\gamma = 63°$ are listed,

$$\begin{aligned} BR(B_d \to D^0\overline{D}^0) &= 2.3 \times 10^{-5}; \\ BR(B_s \to D^0\overline{D}^0) &= 6.8 \times 10^{-4}. \end{aligned} \tag{34}$$

In Fig. 3, we plot the branching ratio of $B_s \to D^0\overline{D}^0$ with different γ. In this figure, we find the branching ratio is not sensitive to CKM angle γ. For the experimental side, there are only upper limits given at 90% confidence level for decay $B_d \to D^0\overline{D}^0$,

$$\begin{aligned} BR(B_d \to D^0\overline{D}^0) &< 6.0 \times 10^{-5}; \qquad BarBar[15] \\ BR(B_d \to D^0\overline{D}^0) &< 4.2 \times 10^{-5}. \qquad Belle[16] \end{aligned} \tag{35}$$

Obviously, our result is consistent with the data. For $B_d \to D^0\overline{D}^0$ decay mode, z_1 is about 6.5, and the strong phase δ_1 is 34°, so A_{CP}^{dir} is about -6% with the definition in Eq.(25). As decay mode $B_s \to D^0\overline{D}^0$ is concerned, z_2 is about 205 and $\delta_2 = 155°$, and the relation between direct CP violation and γ is shown in Fig.4. From the figure, we read the CP asymmetry is about 0.4%, which is rather tiny. It is necessary to state that the z_1 and z_2 are not the true ratio between tree contribution and penguin, because mathematical technics are used in Eq. (20) and (27).

In addition to the perturbative annihilation contributions, there is also a hadronic picture for the $B_d \to D^0\overline{D}^0$, named soft final states interaction[17]. The B meson decays into D^+ and D^-,

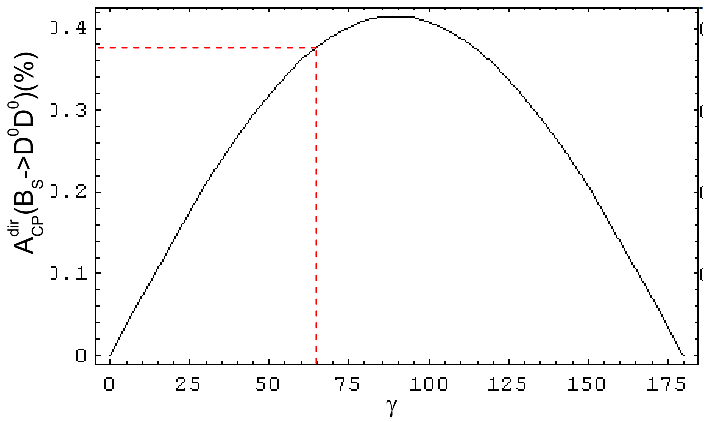

Figure 4: The direct CP-violation of $B_s \to D^0\overline{D}^0$ changes with CKM angle γ.

the secondary particles then exchanging a ρ meson, then scatter into $D^0\overline{D}^0$ through final state interaction afterwards. For B_s decay, the B_s meson decays into D_s^+ and D^+ then scatters into $D^0\overline{D}^0$ by exchanging a Kaon. But this picture cannot be calculated accurately because of lack of many effective vertexes, and we will ignore this contribution here, though it may be important [17].

There are many uncertainties in our calculation such as higher order corrections, the parameters listed in Eq.(33) and the distribution amplitudes of heavy mesons. We will not discuss uncertainty taken by high order correction as we only roughly estimate the branching ratios and CP asymmetries, though high order corrections have been done for some special channels [18, 19] and showed $15-20\%$ uncertainty. The parameters in Eq.(33), fixed by experiments, are proportional to the amplitudes, so we will not analyze this kind uncertainty either. In our calculation, we find that the results are sensitive to the distribution amplitudes, especially to that of D meson. Since the heavy D wave function is less constrained, we set $a_D \in (0.6-0.8)$ GeV and $\omega_D \in (0.35-0.45)$ GeV to exploit the uncertainty. Table 2 shows the sensitivity of the branching ratios to change of ω_b, ω_D and a_D. It is found that uncertainty of the predictions on PQCD is mainly due to ω_D, which describes the behavior in end-point region of D meson, however it is very hard to be determined. Considering the experimental upper limit, our results favor large ω_b, large ω_D and small a_D.

At last, we give the prediction of branching ratios with err bar as follows:

$$\begin{aligned} BR(B_d \to D^0\overline{D}^0) &= (3.8^{+0.5+1.2+0.5}_{-0.6-1.6-0.6}) \times 10^{-5} \left(\frac{f_B \cdot f_D \cdot f_D}{190\text{GeV} \cdot 210\text{GeV} \cdot 210\text{GeV}}\right)^2; \\ BR(B_s \to D^0\overline{D}^0) &= (6.8^{+1.0+2.9+1.0}_{-0.9-2.6-0.9}) \times 10^{-4} \left(\frac{f_{B_s} \cdot f_D \cdot f_D}{230\text{GeV} \cdot 210\text{GeV} \cdot 210\text{GeV}}\right)^2. \end{aligned} \tag{36}$$

Table 2: The sensitivity of the decay branching ratios and CP asymmetries to change of ω_b, ω_D and a_D

	$BR(B_d \to D^0\overline{D}^0)$ $(\times 10^{-5})$	$BR(B_s \to D^0\overline{D}^0)$ $(\times 10^{-4})$	$A_{CP}^{dir}(B_d \to D^0\overline{D}^0)$ (%)	$A_{CP}^{dir}(B_s \to D^0\overline{D}^0)$ (%)
$\omega_b(B \setminus B_s)$				
$0.35 \setminus 0.45$	4.3	7.8	-7.2	0.4
$0.40 \setminus 0.50$	3.8	6.8	-5.3	0.4
$0.45 \setminus 0.55$	3.2	5.9	-5.8	0.4
ω_D				
0.35	5.0	9.7	-4.2	0.3
0.40	3.8	6.8	-5.3	0.4
0.45	2.2	4.2	-7.8	0.5
a_D				
0.6	3.2	5.9	-6.9	0.4
0.7	3.8	6.8	-5.3	0.4
0.8	4.3	7.8	-6.1	0.4

We believe that the $B_d \to D^0\overline{D}^0$ will be measured soon because this ratio is just below the upper limit, and $B_d \to D^0\overline{D}^0$ will be measured in LHC-b in next year as a channel to cross check the γ measurements.

4 Summary

With heavy quark limit and hierarchy approximation $\lambda_{QCD} \ll m_D \ll m_B$, we analyze the $B \to D^0\overline{D}^0$ and $B_s \to D^0\overline{D}^0$ decays, which occur purely via annihilation type diagrams. As a roughly estimation, we calculate the branching ratios and CP asymmetries in PQCD approach. The branching ratios are still sizable. The branching ratio of $B \to D^0\overline{D}^0$ is about 3.8×10^{-5}, which is just below the experimental upper limited result[15, 16], and we think that it will be measured in near future. For $B_s \to D^0\overline{D}^0$, the branching ratio is about 6.8×10^{-4}, which could be measured in LHC-b. From the calculation, it is found that this branching ratio is not sensitive to angle γ. In these two decays, there exist CP asymmetries because of interference between weak and strong interaction, though they are very small.

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